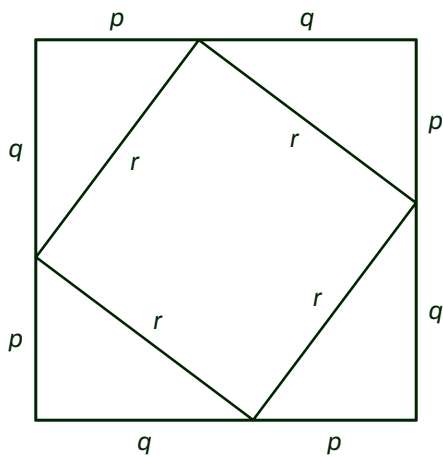


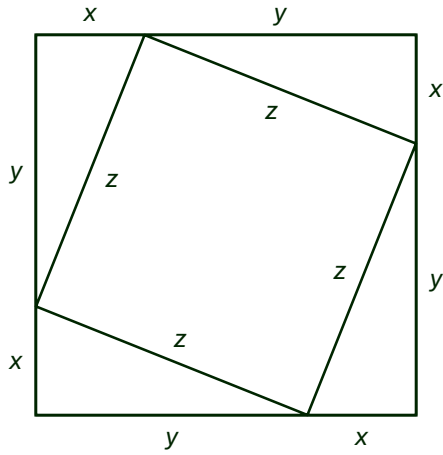
NAME	DATE	SCORE
		/ 10

- 1** The large square below has sides $p + q$. Inside it are four copies of one right triangle, with legs p and q and hypotenuse r . Use the diagram to explain, step by step, why $p^2 + q^2 = r^2$.

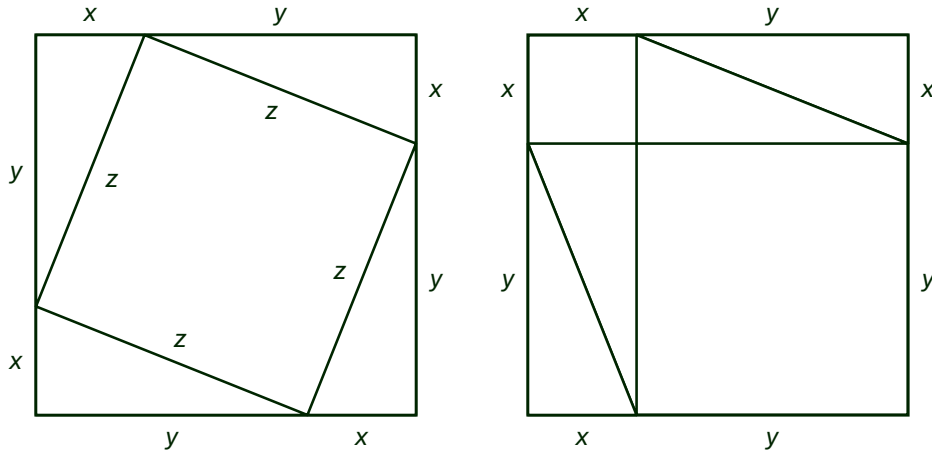


2

The large square below has sides $x + y$. Inside it are four copies of one right triangle, with legs x and y and hypotenuse z . Use the diagram to explain, step by step, why $x^2 + y^2 = z^2$.

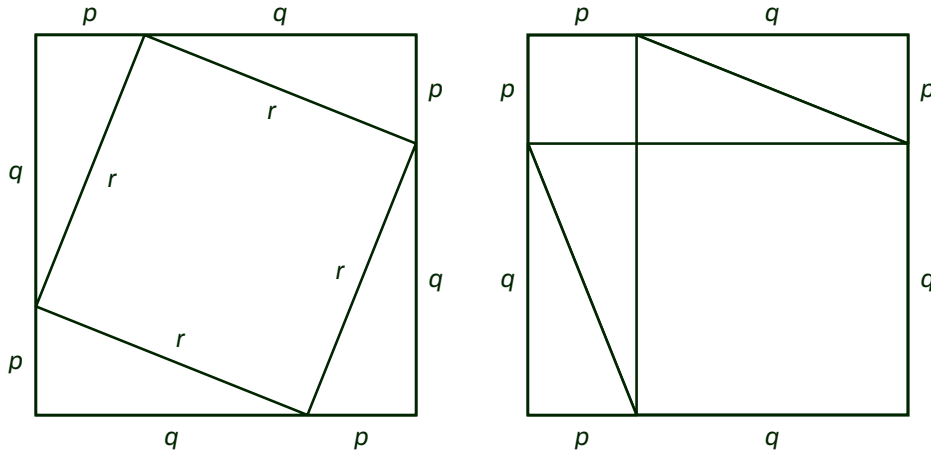


The two large squares below are the same size, with sides $x + y$. Each holds four copies of one right triangle, with legs x and y and hypotenuse z . Use the two diagrams to explain, step by step, why $x^2 + y^2 = z^2$.



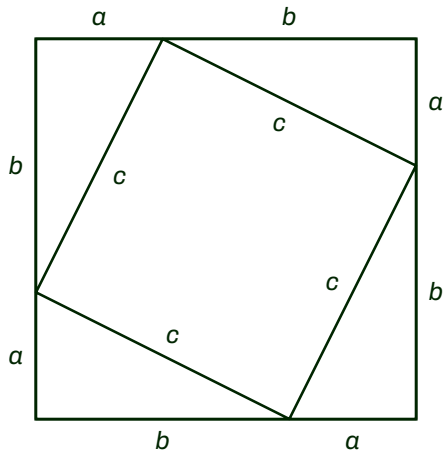
4

The two large squares below are the same size, with sides $p + q$. Each holds four copies of one right triangle, with legs p and q and hypotenuse r . Use the two diagrams to explain, step by step, why $p^2 + q^2 = r^2$.

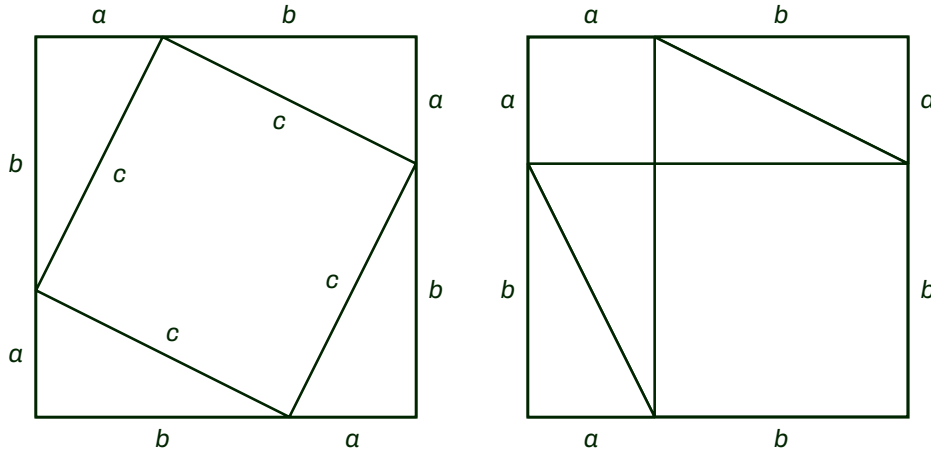


5

The large square below has sides $a + b$. Inside it are four copies of one right triangle, with legs a and b and hypotenuse c . Use the diagram to explain, step by step, why $a^2 + b^2 = c^2$.

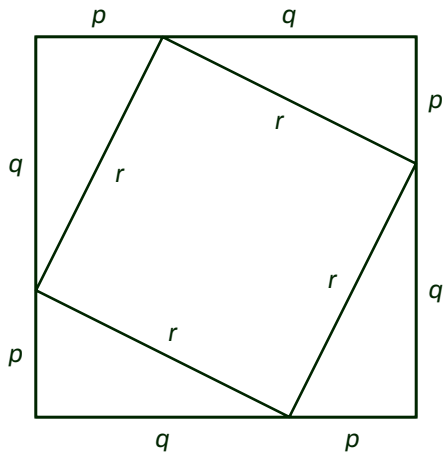


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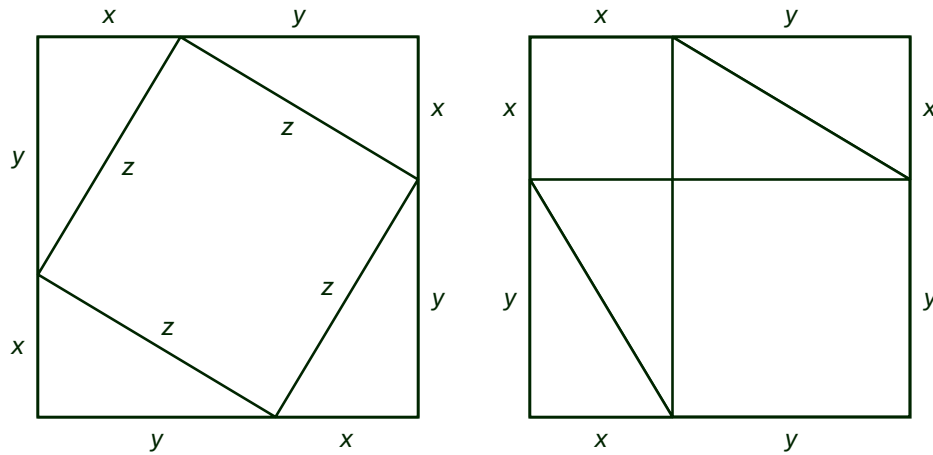
7

The large square below has sides $p + q$. Inside it are four copies of one right triangle, with legs p and q and hypotenuse r . Use the diagram to explain, step by step, why $p^2 + q^2 = r^2$.



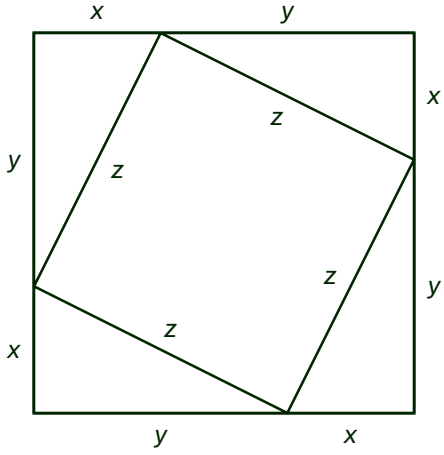
8

The two large squares below are the same size, with sides $x + y$. Each holds four copies of one right triangle, with legs x and y and hypotenuse z . Use the two diagrams to explain, step by step, why $x^2 + y^2 = z^2$.



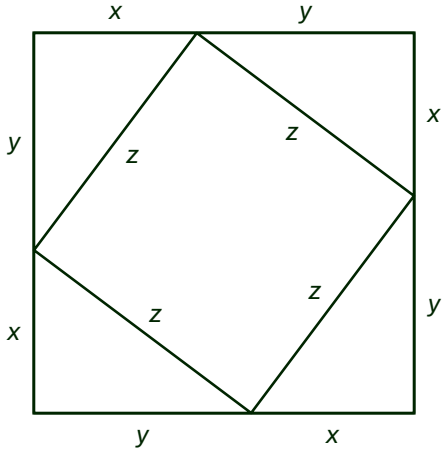
9

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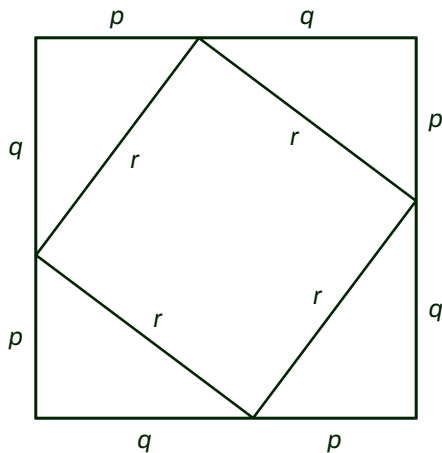
10

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Explain a proof of the Pythagorean theorem

1 A written explanation

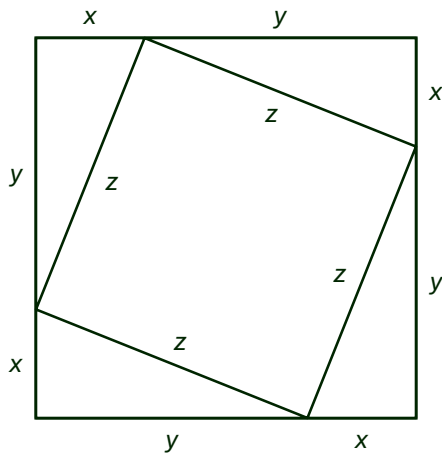


A model answer: The large square has sides $p + q$, so its area is $(p + q)^2 = p^2 + 2pq + q^2$. The same square is made of the four triangles and the square in the middle. Each triangle has area $\frac{1}{2}pq$, so the four make $2pq$; the middle square has sides r , so its area is r^2 . So $p^2 + 2pq + q^2 = 2pq + r^2$, and taking $2pq$ from both sides leaves $p^2 + q^2 = r^2$. (The middle shape is a square: its sides are all r , and at each of its corners the two smaller angles of a triangle, which add to 90° , leave 90° .)

What counts as right: Counts as right if it makes these points: (1) the large square's area is $(p + q)^2$, which is $p^2 + 2pq + q^2$; (2) the same area is the four triangles, $4 \times \frac{1}{2}pq = 2pq$, plus the middle square, r^2 ; (3) setting the two equal and taking away $2pq$ leaves $p^2 + q^2 = r^2$. A stronger answer also says why the middle shape is a square. Checking one triangle with numbers (3, 4 and 5) does not count: the reason must work for every right triangle.

2

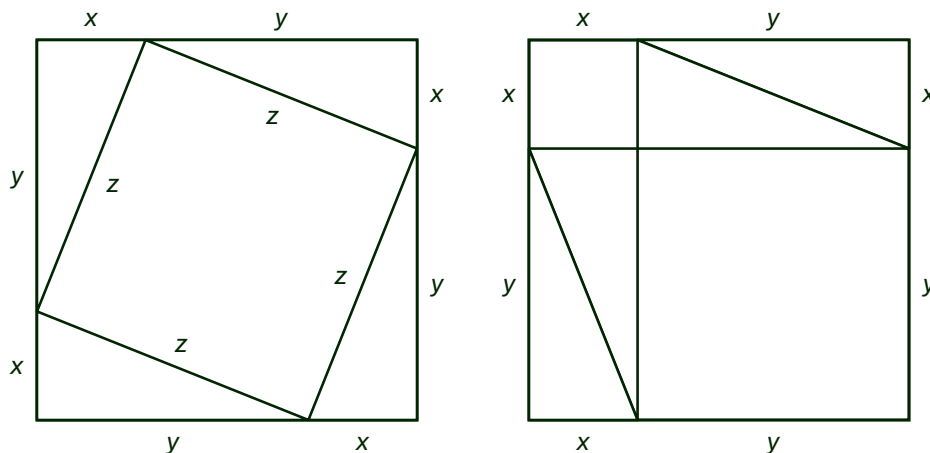
A written explanation



A model answer: The large square has sides $x + y$, so its area is $(x + y)^2 = x^2 + 2xy + y^2$. The same square is made of the four triangles and the square in the middle. Each triangle has area $\frac{1}{2}xy$, so the four make $2xy$; the middle square has sides z , so its area is z^2 . So $x^2 + 2xy + y^2 = 2xy + z^2$, and taking $2xy$ from both sides leaves $x^2 + y^2 = z^2$. (The middle shape is a square: its sides are all z , and at each of its corners the two smaller angles of a triangle, which add to 90° , leave 90° .)

What counts as right: Counts as right if it makes these points: (1) the large square's area is $(x + y)^2$, which is $x^2 + 2xy + y^2$; (2) the same area is the four triangles, $4 \times \frac{1}{2}xy = 2xy$, plus the middle square, z^2 ; (3) setting the two equal and taking away $2xy$ leaves $x^2 + y^2 = z^2$. A stronger answer also says why the middle shape is a square. Checking one triangle with numbers (3, 4 and 5) does not count: the reason must work for every right triangle.

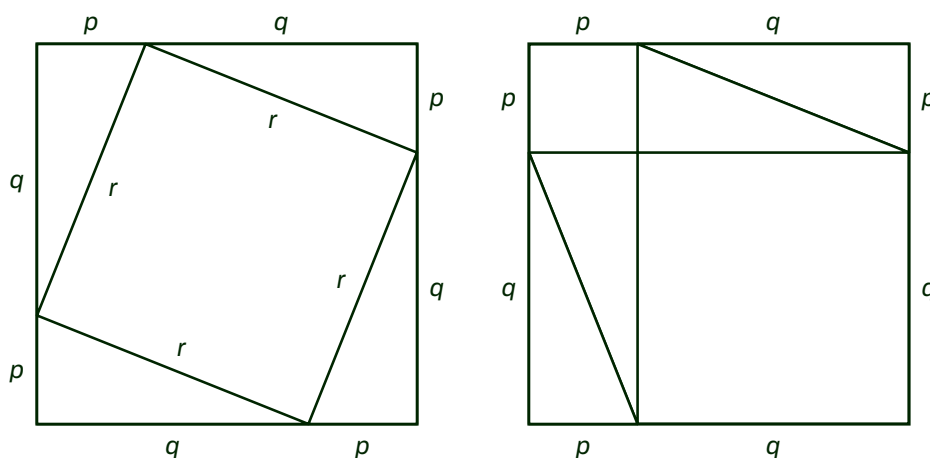
3 A written explanation



A model answer: Both large squares have sides $x + y$, so they have the same area. Each one holds the same four right triangles. Take the four triangles out of each square, and the area left over must be the same. In the first square what is left is one square with sides z , area z^2 . In the second it is two squares, with sides x and y , area $x^2 + y^2$. So $x^2 + y^2 = z^2$.

What counts as right: Counts as right if it makes these points: (1) the two large squares are the same size, so they have the same area; (2) each holds the same four triangles, with the same total area; (3) so what is left when the triangles are taken away has the same area: z^2 in the first square, $x^2 + y^2$ in the second, so $x^2 + y^2 = z^2$. Checking one triangle with numbers (3, 4 and 5) does not count: the reason must work for every right triangle.

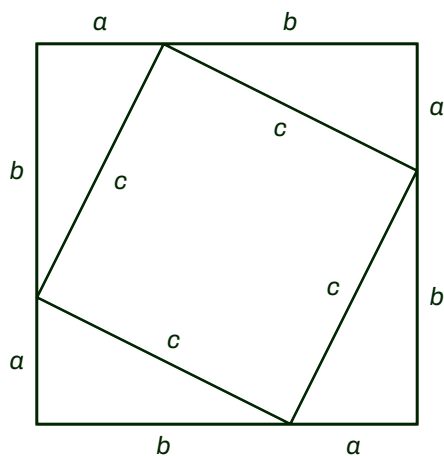
4 A written explanation



A model answer: Both large squares have sides $p + q$, so they have the same area. Each one holds the same four right triangles. Take the four triangles out of each square, and the area left over must be the same. In the first square what is left is one square with sides r , area r^2 . In the second it is two squares, with sides p and q , area $p^2 + q^2$. So $p^2 + q^2 = r^2$.

What counts as right: Counts as right if it makes these points: (1) the two large squares are the same size, so they have the same area; (2) each holds the same four triangles, with the same total area; (3) so what is left when the triangles are taken away has the same area: r^2 in the first square, $p^2 + q^2$ in the second, so $p^2 + q^2 = r^2$. Checking one triangle with numbers (3, 4 and 5) does not count: the reason must work for every right triangle.

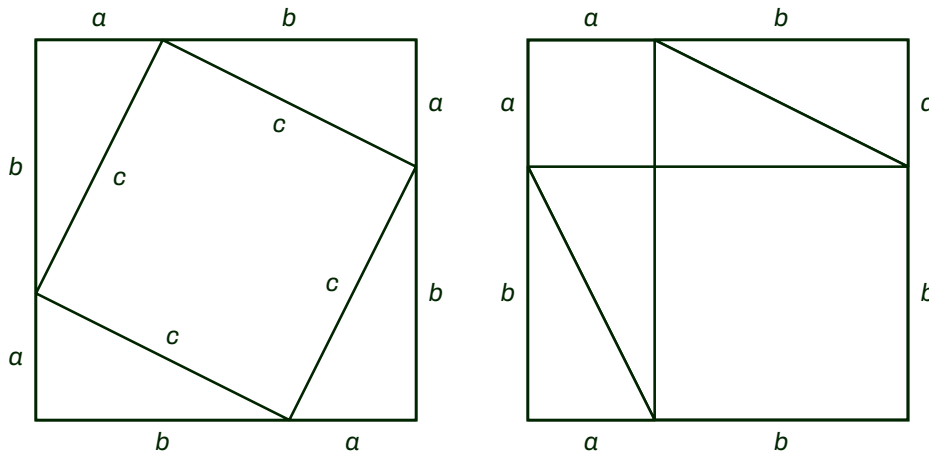
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A model answer: The large square has sides $a + b$, so its area is $(a + b)^2 = a^2 + 2ab + b^2$. The same square is made of the four triangles and the square in the middle. Each triangle has area $\frac{1}{2}ab$, so the four make $2ab$; the middle square has sides c , so its area is c^2 . So $a^2 + 2ab + b^2 = 2ab + c^2$, and taking $2ab$ from both sides leaves $a^2 + b^2 = c^2$. (The middle shape is a square: its sides are all c , and at each of its corners the two smaller angles of a triangle, which add to 90° , leave 90° .)

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6 A written explanation

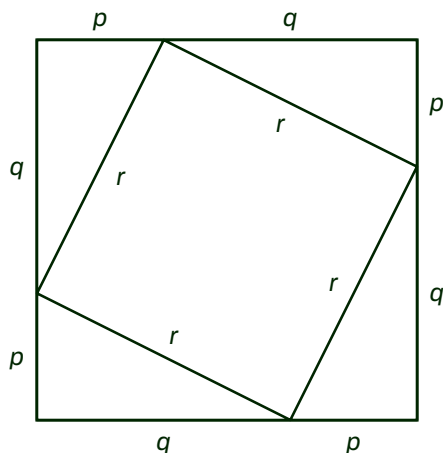


A model answer: Both large squares have sides $a + b$, so they have the same area. Each one holds the same four right triangles. Take the four triangles out of each square, and the area left over must be the same. In the first square what is left is one square with sides c , area c^2 . In the second it is two squares, with sides a and b , area $a^2 + b^2$. So $a^2 + b^2 = c^2$.

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7

A written explanation

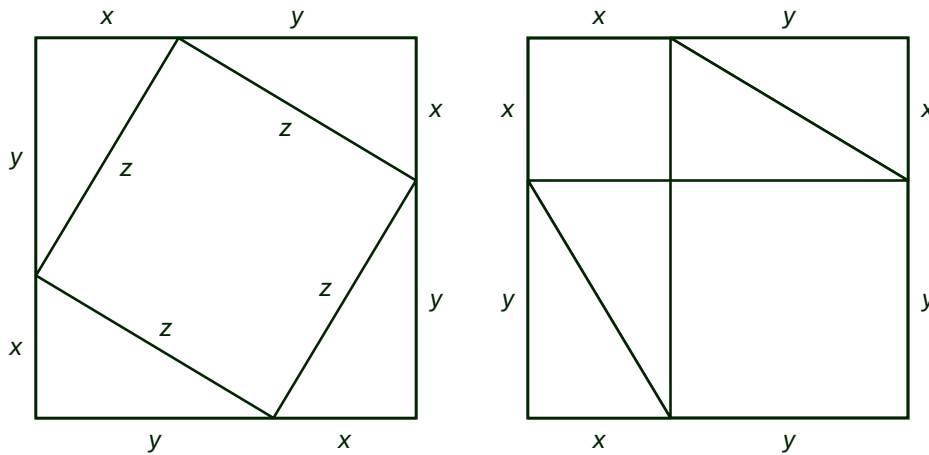


A model answer: The large square has sides $p + q$, so its area is $(p + q)^2 = p^2 + 2pq + q^2$. The same square is made of the four triangles and the square in the middle. Each triangle has area $\frac{1}{2}pq$, so the four make $2pq$; the middle square has sides r , so its area is r^2 . So $p^2 + 2pq + q^2 = 2pq + r^2$, and taking $2pq$ from both sides leaves $p^2 + q^2 = r^2$. (The middle shape is a square: its sides are all r , and at each of its corners the two smaller angles of a triangle, which add to 90° , leave 90° .)

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8

A written explanation

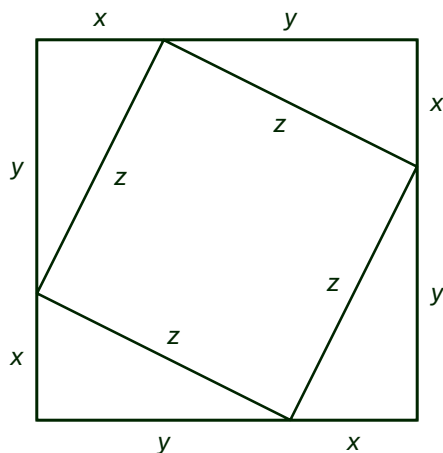


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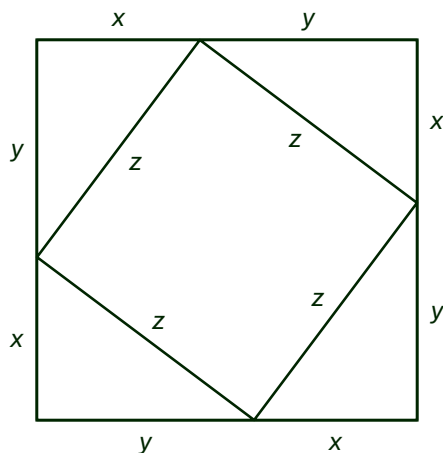
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