

Derive the area of a circle from its circumference

NAME	DATE	SCORE / 10
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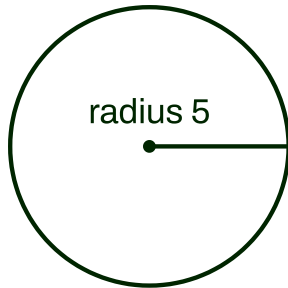
- 1** A circle is cut into many equal slices. The slices are laid side by side, pointing up and down in turn, to make a shape that is nearly a rectangle. The circle has a circumference of 8π centimetres. How tall is this shape, in centimetres?

- 2** A circle is cut into many equal slices. The slices are laid side by side, pointing up and down in turn, to make a shape that is nearly a rectangle. The circle has a radius of 7 centimetres. Why is the area of the circle 49π square centimetres?

- ☐ The shape is 14π centimetres long and 7 centimetres tall
☐ The shape is 7 centimetres long and 7 centimetres tall
☐ The shape is 7π centimetres long and 14 centimetres tall
☐ The shape is 7π centimetres long and 7 centimetres tall

3

A circle is cut into many equal slices. The slices are laid side by side, pointing up and down in turn, to make a shape that is nearly a rectangle. The circle has a radius of 5 centimetres. Explain why the area of the circle is half its circumference times its radius, and so why its area is 25π square centimetres.



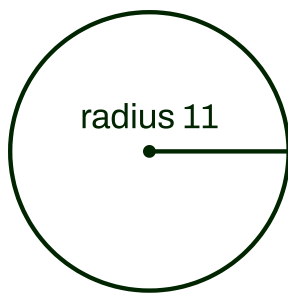
4

A circle is cut into many equal slices. The slices are laid side by side, pointing up and down in turn, to make a shape that is nearly a rectangle. The circle has a diameter of 4 centimetres. The long side of this shape is $\boxed{}\pi$ centimetres. What number goes in the box?

5 A circle is cut into many equal slices. The slices are laid side by side, pointing up and down in turn, to make a shape that is nearly a rectangle. The circle has a radius of 6 centimetres. Why is the area of the circle 36π square centimetres?

- ☐ The shape is 6π centimetres long and 6 centimetres tall
- ☐ The shape is 6 centimetres long and 6 centimetres tall
- ☐ The shape is 12π centimetres long and 6 centimetres tall
- ☐ The shape is 6π centimetres long and 12 centimetres tall

6 A circle is cut into many equal slices. The slices are laid side by side, pointing up and down in turn, to make a shape that is nearly a rectangle. The circle has a radius of 11 centimetres. Explain why the area of the circle is half its circumference times its radius, and so why its area is 121π square centimetres.



7

A circle is cut into many equal slices. The slices are laid side by side, pointing up and down in turn, to make a shape that is nearly a rectangle. The shape is 12π centimetres long and 12 centimetres tall. The area of the circle is $\square\pi$ square centimetres. What number goes in the box?

8

A circle is cut into many equal slices. The slices are laid side by side, pointing up and down in turn, to make a shape that is nearly a rectangle. The circle has a radius of 12 centimetres. Why is the area of the circle 144π square centimetres?

- ☐ The shape is 12π centimetres long and 12 centimetres tall
- ☐ The shape is 24π centimetres long and 12 centimetres tall
- ☐ The shape is 12 centimetres long and 12 centimetres tall
- ☐ The shape is 12π centimetres long and 24 centimetres tall

9

A circle is cut into many equal slices. The slices are laid side by side, pointing up and down in turn, to make a shape that is nearly a rectangle. The circle has a diameter of 20 centimetres. The long side of this shape is $\square\pi$ centimetres. What number goes in the box?

10

A circle is cut into many equal slices. The slices are laid side by side, pointing up and down in turn, to make a shape that is nearly a rectangle. The shape is 4π centimetres long and 4 centimetres tall. The area of the circle is $\square\pi$ square centimetres. What number goes in the box?

Derive the area of a circle from its circumference

1 4

2 The shape is 7π centimetres long and 7 centimetres tall

4 2

5 The shape is 6π centimetres long and 6 centimetres tall

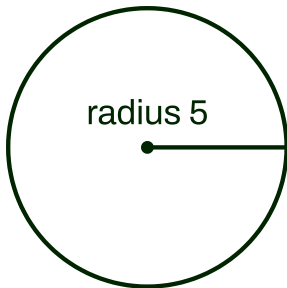
7 144

8 The shape is 12π centimetres long and 12 centimetres tall

9 10

10 16

3 A written explanation

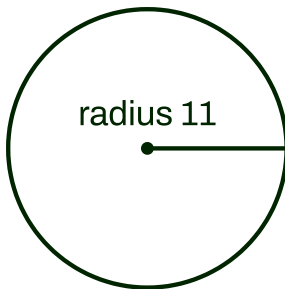


A model answer: The circumference is $2\pi \times 5 = 10\pi$ centimetres. The slices keep all of the circle's area; they are only moved. Half of the curved edge runs along the top of the shape and half along the bottom, so the shape is about half the circumference, 5π centimetres, long. Each slice reaches from the centre to the edge, so the shape is one radius, 5 centimetres, tall. With more and thinner slices the shape gets closer to a rectangle, so the area of the circle is length $\times$ height $= 5\pi \times 5 = 25\pi$ square centimetres. In general that is $\frac{1}{2} \times 2\pi r \times r = \pi r^2$.

What counts as right: Counts as right if it makes these points: (1) cutting and moving the slices keeps the circle's area; (2) the shape is about half the circumference long (5π centimetres), because half the curved edge is along the top and half along the bottom; (3) it is one radius tall (5 centimetres), the length of a slice; (4) so the area is half the circumference times the radius, $5\pi \times 5 = 25\pi$ square centimetres, which is $\pi \times r \times r$. Only putting 5 into $A = \pi r^2$ does not count: that uses the formula instead of explaining it.

6

A written explanation



A model answer: The circumference is $2\pi \times 11 = 22\pi$ centimetres. The slices keep all of the circle's area; they are only moved. Half of the curved edge runs along the top of the shape and half along the bottom, so the shape is about half the circumference, 11π centimetres, long. Each slice reaches from the centre to the edge, so the shape is one radius, 11 centimetres, tall. With more and thinner slices the shape gets closer to a rectangle, so the area of the circle is length $\times$ height = $11\pi \times 11 = 121\pi$ square centimetres. In general that is $\frac{1}{2} \times 2\pi r \times r = \pi r^2$.

What counts as right: Counts as right if it makes these points: (1) cutting and moving the slices keeps the circle's area; (2) the shape is about half the circumference long (11π centimetres), because half the curved edge is along the top and half along the bottom; (3) it is one radius tall (11 centimetres), the length of a slice; (4) so the area is half the circumference times the radius, $11\pi \times 11 = 121\pi$ square centimetres, which is $\pi \times r \times r$. Only putting 11 into $A = \pi r^2$ does not count: that uses the formula instead of explaining it.